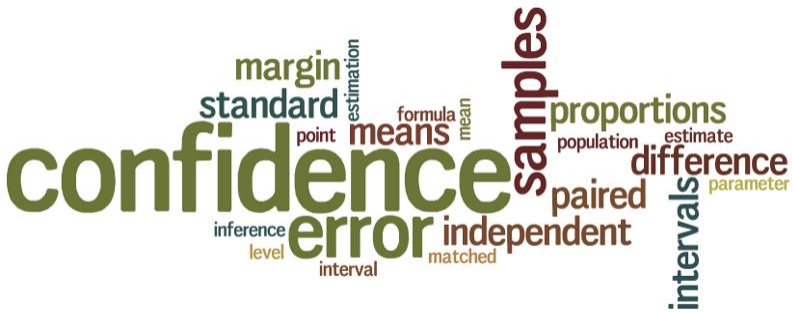
ANATOMY OF A CONFIDENCE INTERVAL



OVERVIEW OF CONFIDENCE INTERVALS

Preface: The term confidence interval is commonly abbreviated (CI). The symbol (±) is read as "plus or minus". The discussion below employs a degree of intentional redundancy for those not versed in the subject matter.

What is a confidence interval?

Answer: A confidence interval is an estimation.

A confidence interval estimates a range of values that contains/captures the true population value for a variable of interest. Implicit in this concept is a level of confidence that this actual/true population value is contained within the estimated range of values.

A confidence interval uses information from a sample (aka sample data) gathered from a larger population. From a purely mathematical viewpoint, ideally this sample has the characteristic of randomness and is known as a random sample. However, in practice, true randomness is difficult to achieve. Accordingly, the goal is to have a sample that is representative of the population of interest. Hence, the goal is to obtain a representative sample.

The information from this representative sample is used to calculate an estimated value and a surrounding range of values for a variable of interest.

BREAKING DOWN CONFIDENCE INTERVALS

Informally, a confidence interval (CI) is a range of values (aka an interval) which is likely to contain a population value (aka a parameter) for "something" that is being estimated. This interval contains/surrounds what is known as an estimate (aka a point estimate). Ideally, a narrow confidence interval (CI) if desirable because it conveys a better (more accurate) estimation of the actual population value.

The "something" being estimated may include:

- **population proportions**
- **population means (arithmetic averages)**
- **differences between population means or proportions**
- **estimates of variation among groups**

Constructing a CI begins by calculating a point estimate from the sample data. However, a point estimate doesn't provide any information about the variation around this "average" value. Accordingly, confidence intervals are used to communicate variation around a point estimate. This variation is measured via an interval of values known in statistical jargon as the margin of error.

Hence, a confidence consists of two parts…

THE POINT ESTIMATE & THE MARGIN OF ERROR

Remember, both the point estimate and the margin of error are derived from the sample data.

THE POINT ESTIMATE

A point estimate is the "average" value calculated from the sample data.

Examples of everyday point estimates:

- **From a representative sample, a person's average height can be used to estimate the average height of a larger population -- The average height of men in the US is 5 feet 11 inches.**
- **From a representative sample, the average proportion of voters favoring a ballot proposal can be used to estimate the proportion of a larger population -- 68% of Michigan voters favor proposal A.**

THE MARGIN OF ERROR

A margin of error is a range of values calculated from the sample data; the point estimate is contained within this interval.

Examples of margin of error estimates:

- **The range of heights (from a representative sample) can be used to estimate the average range of heights of a larger population -- The average height of men in the US is between 5 feet 8 inches and 6 feet 2 inches. The margin of error is 6 inches (6' 2" inches minus 5' 8" inches).**
- **The range of the proportion of voters favoring a ballot proposal (from a representative sample) can be used to estimate the range of the proportions of a larger population -- The percentage of Michigan voters who favor proposal A is between 64% and 72%. The margin of error is 8% (72% minus 64%).**

The margin of error consists of two parts:

1. **A standard error**
2. **A distribution multiplier**

THE STANDARD ERROR

The standard error (SE) is related to standard deviation of a dataset. The standard deviation tells you how spread out the data are. It is a measure of how far observed values are from the center (average) of the population distribution

The standard error is very similar to standard deviation. Both are measures of spread / dispersion. The more spread out the data are, the greater the standard error and the greater the standard deviation.

When conducting research, you often only collect data for a small sample of the entire population. This sample has an error associated with it. This occurs because each sample will be slightly different and each will produce slightly different estimates.

The standard error measures how different a sample estimate is likely to be from a population value. It tells you how the estimate would vary if you were to repeat a study using different samples (of the same size) taken from a single population. In other words, it quantifies the difference between a parameter (i.e., a population value) and a statistic (i.e., a sample value).

The most important connection/relationship between data and standard errors is …

The Larger the Sample Size… the Smaller the Standard Error

THE DISTRIBUTION MULTIPLIER

- **most often referred to as just the multiplier**
- **occasionally called a t-multiplier or z-multiplier**
- **sometimes termed a critical value**

The multiplier is a function of the underlying population and the size/width of the confidence interval. Typical confidence intervals range from 50% to 99.9%, with the most common being 95%.

The concept of the multiplier is more easily explained and understood by looking at an example containing specific multiplier values used to generate a margin of error.

The table below lists six multipliers for normal distributions (aka z distributions) and their corresponding confidence levels. (These "z multipliers" are used when an assumption of an underlying normal population distribution is satisfied.)

Confidence Level	z multiplier
80%	1.282
90%	1.645
95%	1.960

Confidence Level	z multiplier
99%	2.576
99.5%	2.807
99.9%	3.291

Please note, that as confidence levels increase, so too do the magnitudes of the multipliers. These multiplier values are multiplied by the standard error value to generate the margin of error; hence, larger multipliers will increase the magnitude/size of the margin of error. Recall, that the overall confidence interval is calculated by "adding" and "subtracting" (±) the margin of error to the point estimate.

Consequently, the greater the confidence level, the greater the multiplier, the greater the margin of error, and the greater/wider the associated confidence interval.

Confidence Level ⇧ ➡ Multiplier ⇧ ➡ Margin of Error ⇧ ➡ Confidence Interval range/size ⇧

Pulling everything together…

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SUMMARIZING THE ANATOMY OF A CONFIDENCE INTERVAL

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A confidence interval is composed of two parts…

POINT ESTIMATE ± MARGIN OF ERROR

Additionally, the MARGIN OF ERROR consists of two parts:

- **the multiplier**
- **the standard error**

Accordingly, a formal definition of confidence interval is…

POINT ESTIMATE ± (multiplier * standard error)

These two examples may help to solidify the concept of a confidence interval:

Example #1

You survey 100 people about their television-viewing habits, and find they view an average of 35 hours of television per week. The survey has a margin of error of 1.4 hours.

Point estimate is 35 hours. Margin of error is 1.4 hours

CI is -- 35 hours ± 1.4 hours

This confidence interval is reported as: 95% CI [33.6, 36.4] *

Example #2

Based on a representative sample, on average, 52% of all births are female. The sample has a margin of error of 4% of births.

Point estimate is 52% of births. Margin of error is 4% of births.

CI is -- 52% of births ± 4% of births

This confidence interval is reported as: 95% CI [0.48, 0.56] *

***APA Style -- Reporting results may differ and are dependent on various writing style requirements.**